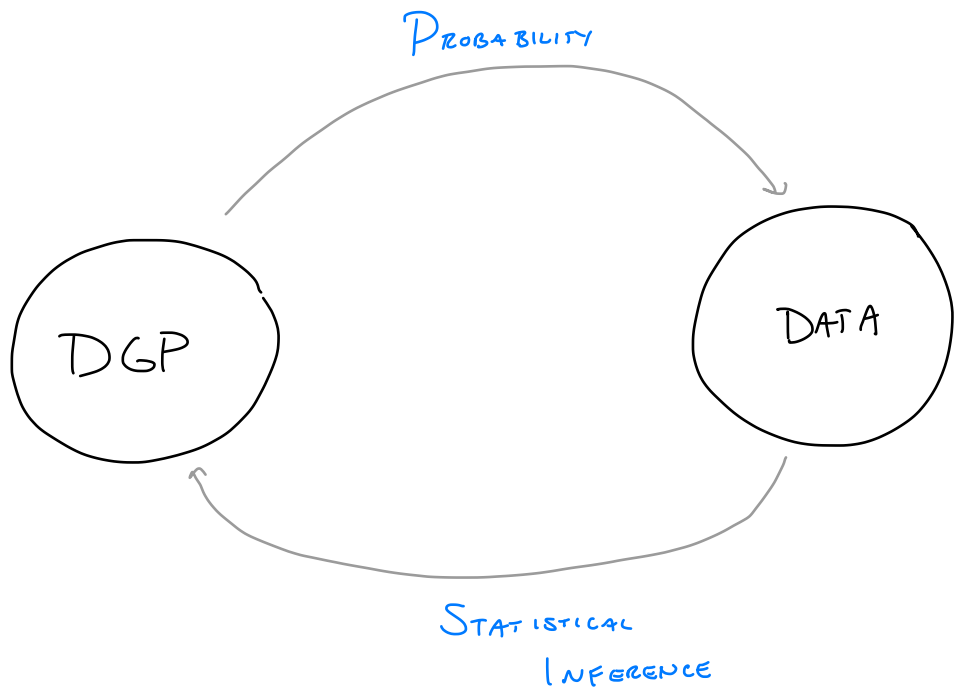


ALL OF STATISTICS

STAT 510



WHAT IS STATISTICS?

THE STUDY OF STATISTICS!

WHAT IS A STATISTIC?

A FUNCTION OF DATA!

TWO VIEWS OF A STATISTIC

DATA: x_1, x_2, \dots, x_n

DATA ANALYSIS

$$T(x_1, x_2, \dots, x_n) = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x}$$

SAMPLE STATISTIC

$$T(X_1, X_2, \dots, X_n) = \frac{1}{n} \sum_{i=1}^n X_i = \bar{X}$$

RANDOM VARIABLE

MATHEMATICAL STATISTICS

RANDOM SAMPLES

$$X_1, X_2, \dots, X_n \sim F$$

WANT TO LEARN

$$X_1, X_2, \dots, X_n \sim N(\mu, \sigma^2)$$

WANT TO LEARN

STATISTICS AS ESTIMATORS

θ UNKNOWN PARAMETER

$$\hat{\theta}_n = g(x_1, x_2, \dots, x_n)$$

ESTIMATOR

STATISTIC USED AS A "GOOD GUESS" FOR θ .

$$\theta = T(F)$$

PARAMETER : DISTRIBUTION :: ESTIMATION : DATA

$$\hat{\theta}_n = T(\hat{F}_n)$$

EMPIRICAL CDF

How Good is an Estimator?

- Bias
- Variance
- $MSE = \text{BIAS}^2 + \text{VAR}$

IN GENERAL

- LOSS
- RISK
- ADMISSABILITY

How TO CREATE ESTIMATORS ?

- PLUG-IN PRINCIPLE
- MAXIMUM LIKELIHOOD
- METHOD OF MOMENTS
- BAYESIAN INFERENCE

WHAT CAN WE DO WITH ESTIMATORS?

- POINT ESTIMATES
- INTERVAL ESTIMATES
- HYPOTHESIS TESTS

A NOTE ON STANDARD ERRORS

USE $\hat{\theta}$ TO ESTIMATE θ

WANT CI: $\hat{\theta} \pm \hat{SE}(\hat{\theta})$

$SE(\hat{\theta}) = E[\hat{\theta}]$ ← DEPENDS ON DISTRIB OF $\hat{\theta}$
 $SE(\hat{\theta}) = SD[\hat{\theta}]$ ← DEPENDS ON DISTRIB OF $\hat{\theta}$
 $\hat{SE}(\hat{\theta}) = \hat{SD}[\hat{\theta}]$ ← ALSO DEPENDS ON ESTIMATES OF PARAMS

$$E[\bar{x}] = \mu$$

$$SD[\bar{x}] = \sigma/\sqrt{n}$$

$$\hat{SD}[\bar{x}] = \hat{\sigma}/\sqrt{n}$$

WHAT IS MATHEMATICAL STATISTICS?

- DEVELOP NEW STATISTICS (METHODS)
- UNDERSTAND THEIR PROPERTIES
- ARGUE FOR USE IN PRACTICE