Likelihood Ratio Test

Stat 510

$$X \sim \text{BINOM}(\overset{\text{KNOWN}}{n}, p)$$

$\hat{p} = \frac{x}{n}$ MLE (ALTERNATIVE)

$H_0: \; p = p_0$

$H_1: \; p \neq p_0$

NULL $\longrightarrow \mathcal{L}(p_0) = \binom{n}{x} p_0^{x} (1 - p_0)^{n-x}$

ALT $\longrightarrow \mathcal{L}(\hat{p}) = \binom{n}{x} \hat{p}^{x} (1 - \hat{p})^{n-x}$

$$\Lambda = \frac{\mathcal{L}(\hat{p})}{\mathcal{L}(p_0)} = \left(\frac{\hat{p}}{p_0}\right)^x \left(\frac{1-\hat{p}}{1-p_0}\right)^{n-x}$$

REJECT WHEN "LARGE"

$$\lambda = 2\log\Lambda = 2\log\left(\frac{\hat{p}}{p_0}\right) + 2(n-x)\log\left(\frac{1-\hat{p}}{1-p_0}\right)$$

FOR LARGE SAMPLES

$$\lambda \overset{\text{APPROX}}{\sim} \chi^2_1$$

$$\text{p-VALUE} = P(\chi^2_1 > \lambda)$$

$$X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$$

BOTH UNKNOWN

$H_0 : \mu = \mu_0$ $\quad \theta_0 = (\mu_0, \sigma_0^2)$

$H_1 : \mu \neq \mu_0$ $\quad \theta_1 = (\mu_1, \sigma_1^2)$

NEED TO ESTIMATE

$$\hat{\sigma}_0^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_0)^2$$

$$\hat{\theta}_0 = (\mu_0, \hat{\sigma}_0^2)$$

$$\hat{\theta}_1 = (\hat{\mu}_1, \hat{\sigma}_1^2)$$

MLE (ALT)

$$\hat{\mu}_1 = \bar{x}$$

$$\hat{\sigma}_1^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\mathcal{L}(\theta) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \operatorname{Exp}\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

$$\mathcal{L}(\hat{\theta}_1) = \left(\frac{1}{2\pi\hat{\sigma}_1^2}\right)^{n/2} \operatorname{Exp}\left[-\frac{1}{2\hat{\sigma}_1^2}\sum_{i=1}^{n}(x_i-\bar{x})^2\right] = \left(\frac{n e^{-1}}{2\pi\sum_{i=1}^{n}(x_i-\bar{x})^2}\right)^{n/2}$$

$$\mathcal{L}(\hat{\theta}_0) = \left(\frac{1}{2\pi\hat{\sigma}_0^2}\right)^{n/2} \operatorname{Exp}\left[-\frac{1}{2\hat{\sigma}_0^2}\sum(x_i-\mu_0)^2\right] = \left(\frac{n e^{-1}}{2\pi\sum_{i=1}^{n}(x_i-\mu_0)^2}\right)^{n/2}$$

$$\Lambda = \frac{\mathcal{L}(\hat{\theta}_1)}{\mathcal{L}(\hat{\theta}_0)} = \left(\frac{\sum(x_i-\mu_0)^2}{\sum(x_i-\bar{x})^2}\right)^{n/2}$$