Moments

Stat 510

DEFN

THE k^{th} MOMENT OF X IS DEFINED TO BE

$$\mathbb{E}\left[X^k\right]$$

ASSUMING IT EXISTS, THAT IS

$$\mathbb{E}\left[|X|^k\right] < \infty$$

DEFN

THE MOMENT GENERATING FUNCTION, MGF, OF X IS

$$\psi_X(t) = \mathbb{E}\left[e^{tX}\right] = \int e^{tx}\, dF(x)$$

NOTE THAT

$$\psi'(0) = \left[\frac{d}{dt}\mathbb{E}\left[e^{tX}\right]\right]_{t=0} = \mathbb{E}\left[\frac{d}{dt}e^{tX}\right]_{t=0} = \mathbb{E}\left[Xe^{tX}\right]_{t=0} = \mathbb{E}[X]$$

IN GENERAL

$$\psi^{(k)}(0) = \mathbb{E}\left[X^k\right]$$

MGF Properties

(1) $Y = aX + b \implies \psi_Y(t) = e^{bt}\psi_X(at)$

(2) $X_1 \dots X_n$ IND AND $Y = \sum_{i=1}^{n} X_i$

$$\implies \psi_Y(t) = \prod_{i=1}^{n} \psi_{X_i}(t)$$

WHERE $\psi_{X_i}(t)$ IS THE MGF OF X_i

INSERT JOKE HERE

THM X, Y RVs

IF $\varphi_X(t) = \varphi_Y(t)$ FOR ALL t IN AN OPEN INTERVAL AROUND 0

THEN $X \overset{d}{=} Y$.

RECALL, DOES NOT MEAN $X = Y$.

Example

$X_1 \sim \text{Pois}(\lambda_1)$

$X_2 \sim \text{Pois}(\lambda_2)$

X_1, X_2 IND

$Y = X_1 + X_2 \sim ?$

$$\psi_{X_1}(t) = e^{\lambda_1(e^t - 1)}$$

$$\psi_{X_2}(t) = e^{\lambda_2(e^t - 1)}$$

$$\psi_Y(t) = e^{\lambda_1(e^t-1)} e^{\lambda_2(e^t-1)}$$

$$= e^{(\lambda_1+\lambda_2)(e^t-1)}$$

MGF OF $\text{Pois}(\lambda_1 + \lambda_2)$

Thus $Y \sim \text{Pois}(\lambda_1 + \lambda_2)$