

Two or more RANDOM VARIABLES

STAT 510

JOINT DISTRIBUTIONS

X, Y DISCRETE RVs

JOINT PMF

$$f_{X,Y}(x,y) = P[X=x, Y=y]$$

"AND"

X, Y CONTINUOUS RVs

JOINT PDF $f_{X,Y}(x,y)$

JOINT CDF

- $f(x,y) \geq 0$

- $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dx dy = 1$

- $P((X,Y) \in A) = \iint_A f(x,y) dx dy$

$$F_{X,Y}(x,y) = P[X \leq x, Y \leq y]$$

MARGINAL DISTRIBUTIONS

X, Y JOINT $f(x, y)$

DISCRETE

$$f_X(x) = P[X=x] = \sum_y P[X=x, Y=y] = \sum_y f(x, y)$$

$$f_Y(y) = P[Y=y] = \sum_x P[X=x, Y=y] = \sum_x f(x, y)$$

CONTINUOUS

$$f_X(x) = \int f(x, y) dy$$

$$f_Y(y) = \int f(x, y) dx$$

INDEPENDENT RANDOM VARIABLES

DEFN X, Y IND IF

$$P(X \in A, Y \in B) = P(X \in A)P(Y \in B) \quad \text{FOR ALL } A, B$$

Thm X, Y JOINT PDF $f_{X,Y}(x,y)$

$$X \text{ IND } Y \iff f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

FOR ALL x, y

CONDITIONAL DISTRIBUTIONS

X, Y DISCRETE

CONDITIONAL PMF

IF $f_Y(y) > 0$

$$f_{X|Y}(x|y) = \mathbb{P}[X=x | Y=y]$$

↑ INPUT TO FUNCTION

$$= \frac{\mathbb{P}[X=x | Y=y]}{\mathbb{P}[Y=y]}$$

↑ FIXED

$$= \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

↑ JOINT

↑ MARGINAL

CONDITIONAL PDF

$f_Y(y) > 0$

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

JOINT

TOGETHER



IND?

MARGINAL

ALONE



CONDITIONAL

GIVEN



RANDOM VECTORS

$$X = (X_1, X_2, \dots, X_n)$$

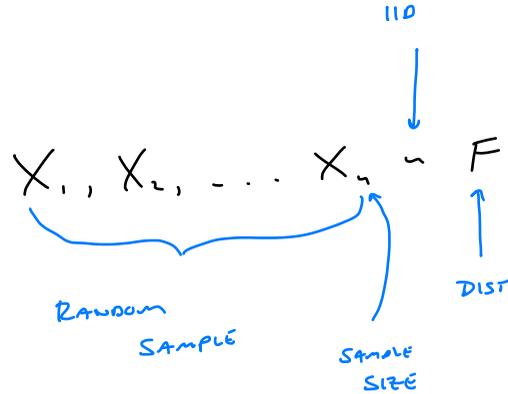
RANDOM VARIABLES

$$f(x_1, x_2, \dots, x_n) \quad \text{JOINT PDF}$$

CAN GET MARGINALS AND CONDITIONALS

IID RANDOM VARIABLES

INDPENDENT
IDENTICALLY
DISTRIBUTED



$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^n f_{X_i}(x_i) = \prod_{i=1}^n f(x_i)$$

↑ IND

↑ IDENT

TRANSFORMATIONS

$$X_1 \sim \text{Pois}(\lambda_1) \quad X_2 \sim \text{Pois}(\lambda_2) \quad Y = X_1 + X_2 \sim \text{Pois}(\lambda_1 + \lambda_2)$$

LOOKUP

$$X_1, X_2, \dots, X_n \text{ i.i.d. } \mathcal{N}(\mu, \sigma^2)$$

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

MEMORIZE

$$Y = f(X_1, X_2) \sim ?$$

- CDF
- CHANGE OF VARIABLES
- MGF