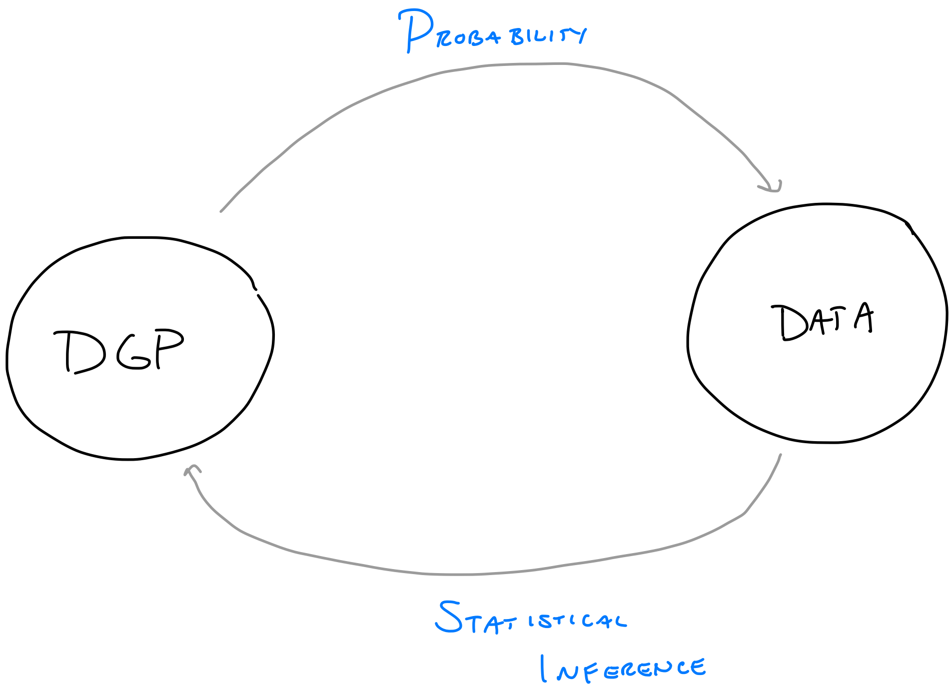
PROBABILITY

STAT 510


PROBABILITY
DGP
DATA
STATISTICAL INFERENCE

"DEFINITIONS ARE YOUR FRIENDS!"

— MARIUS JUNGE

MATH 347

FALL 2005

DEFINITION

A FUNCTION $\mathbb{P}$ IS A PROBABILITY DISTRIBUTION IF

1. $\mathbb{P}(A) \geq 0$ FOR ALL A

2. $\mathbb{P}(\Omega) = 1$

   SAMPLE SPACE

3. $A_1, A_2, A_3, \ldots$ DISJOINT

   THEN $\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$

INTERPRETATION ?

"LONG RUN" PROPORTION ? → FREQUENTIST

DEGREE OF BELIEF ? → BAYESIAN

$\mathbb{P}$(COIN LANDS HEADS) = ?

$\mathbb{P}$(CUBS WIN 2021 WORLD SERIES) = ?

PROBABILITY PROPERTIES

EMPTY SET

- $\mathbb{P}(\phi) = 0$
- $A \subset B \implies \mathbb{P}(A) \leq \mathbb{P}(B)$
- $0 \leq \mathbb{P}(A) \leq 1$

COMPLEMENT

- $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$

$\mathbb{P}(A \cap B)$

- $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(AB)$

"OR" "AND"

- $A \cap B = \phi \implies \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

"DISJOINT"

Defn

Events A, B are independent if

$$\mathbb{P}(AB) = \mathbb{P}(A)\mathbb{P}(B)$$

Two uses

- Assuming
- Checking

DEFN

GIVEN $\mathbb{P}(B) > 0$ THEN THE CONDITIONAL PROBABILITY

OF A GIVEN B IS

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(AB)}{\mathbb{P}(B)}$$

"GIVEN"

- $\mathbb{P}(AB) = \mathbb{P}(B)\,\mathbb{P}(A \mid B)$ MULTIPLICATION RULE
- $\mathbb{P}(A \mid B) \neq \mathbb{P}(B \mid A)$ IN GENERAL
- $\mathbb{P}(\cdot \mid B)$ SATISFIES THE AXIOMS FOR FIXED B
- $\mathbb{P}(A \mid \cdot)$ DOES NOT

A, B INDEPENDENT IF AND ONLY IF

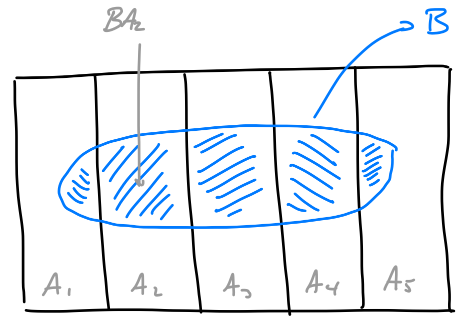
$$\mathbb{P}(A) = \mathbb{P}(A \mid B)$$

$$\mathbb{P}(AB) = \mathbb{P}(A \mid B)\,\mathbb{P}(B)$$
$$= \mathbb{P}(A)\,\mathbb{P}(B)$$

THM LAW OF TOTAL PROBABILITY

LET $A_1, A_2, \ldots, A_k$ BE A PARTITION OF Ω

- DISJOINT
- $\bigcup_{i=1}^{\infty} A_i = \Omega$



$$\mathbb{P}(B) = \sum_{i=1}^{k} \mathbb{P}(BA_i) = \sum_{i=1}^{k} \mathbb{P}(B|A_i)\,\mathbb{P}(A_i)$$

$B = \bigcup_{i=1}^{k} BA_i$, BA_i DISJOINT

THM BAYES' THEOREM

"FLIP THE CONDITIONAL"

LET $A_1, A_2, \ldots, A_k$ BE A PARTITION OF Ω

$P(A_i) > 0$ FOR EACH i

$P(B) > 0$

POSTERIOR

CONDITIONAL PROB

MULTIPLICATION RULE

$$P(A_j \mid B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B \mid A_j) P(A_j)}{P(B)}$$

PRIOR

$$= \frac{P(B \mid A_j) P(A_j)}{\sum_{i=1}^{k} P(B \mid A_i) P(A_i)}$$

BAYES' THM

LAW OF TOTAL PROB

Example

$\mathbb{P}(D) = 0.025 \qquad \mathbb{P}(+ \mid D) = 0.94 \qquad \mathbb{P}(+ \mid D') = 0.04$

DISEASE POSITIVE TEST

$\mathbb{P}(D \mid +) = ?$

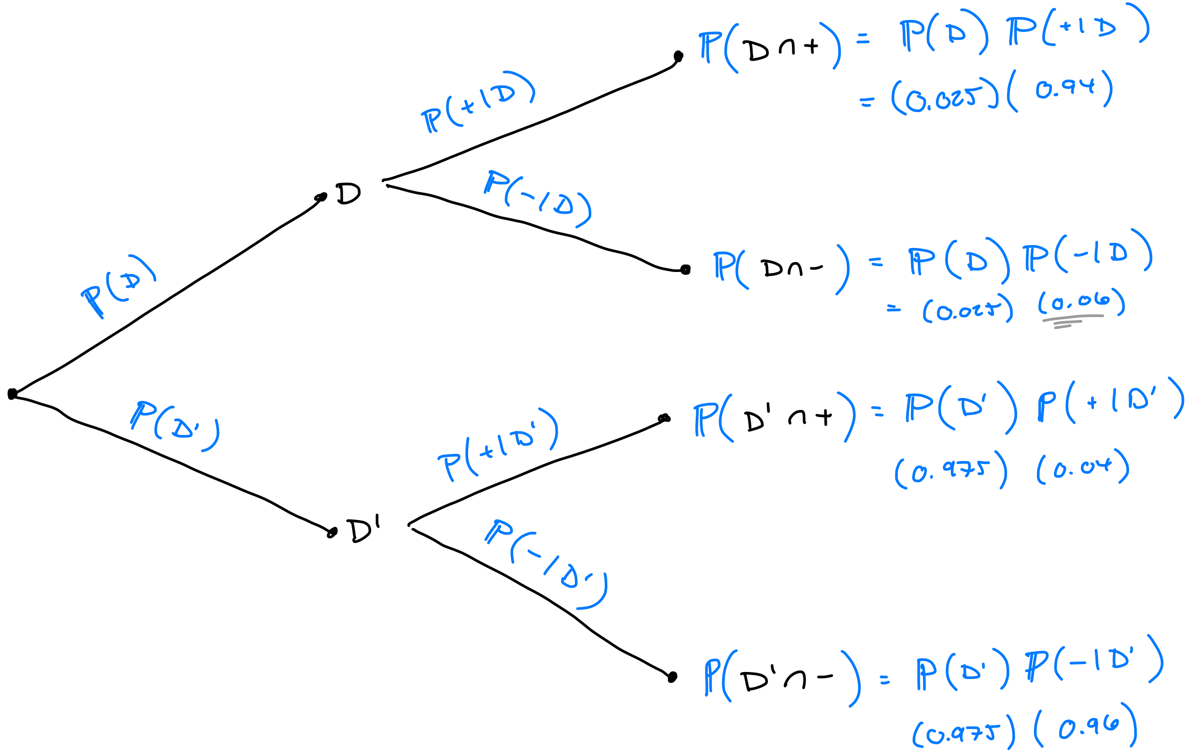
$\mathbb{P}(D' \mid -) = ?$

NO DISEASE NEGATIVE TEST

$$\mathbb{P}(D \mid +) = \frac{\mathbb{P}(+ \mid D)\,\mathbb{P}(D)}{\mathbb{P}(+ \mid D)\,\mathbb{P}(D) + \mathbb{P}(+ \mid D')\,\mathbb{P}(D')}$$

$$= \frac{(0.94)(0.025)}{(0.94)(0.025) + (0.04)(0.975)}$$

$$= 0.376 \;!!!$$



$$\mathbb{P}(D' \mid -) = \frac{\mathbb{P}(D' \cap -)}{\mathbb{P}(-)} = \frac{\mathbb{P}(D' \cap -)}{\mathbb{P}(D' \cap -) + \mathbb{P}(D \cap -)} = 0.9984$$